Resonance frequency analysis of MEMS based on piezoelectric micro-cantilever beam for vibration energy harvesting applications

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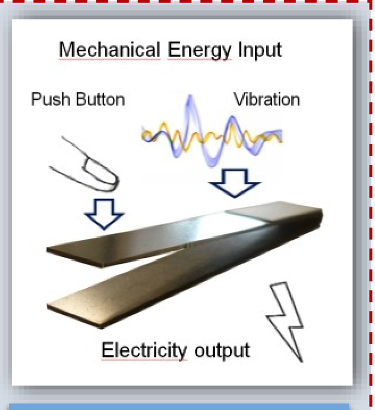
Introduction

Microfabricated micro cantilevers are widely used as a nanomechanical tool for diagnostics, molecular detections and many sensor devices. Its easiness, sensivity and cost of manufacture makes it a key element for many applications.

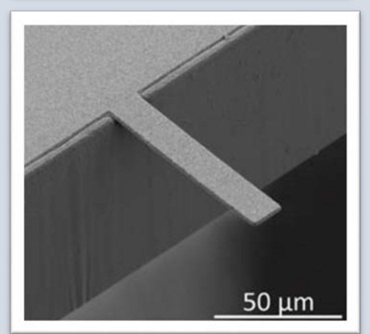
Piezoelectric material covered unimorph microcantilevers would provide energy generation from ambient sources as a result of its freely vibrating body. To generate maximum amount of energy, the microcantilever structure's resonance frequency (natural frequency) should have be specifically adjusted for energy sources. However resonance frequency is limited by cantilever's dimensions and materials.

Consciously designed cantilevers will be able to harvest maximum amount of energy from ambient sources. Hence resonance frequency of double layered unimorph cantilever should have be investigated.

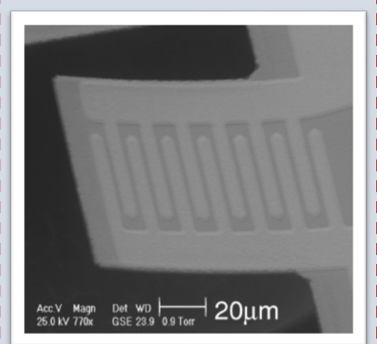
In this study we have represented and demonstrated the finite element method (FEM) to understand the effect of thickness and length parameters on resonance frequencies of silicon based on microcantilever beam. A thin layer of Lead Zirconate Titanate (PZT-5H) was integrated on rectangular silicon proof mask cantilever as piezoelectric material. COMSOL Multiphysics which is commercial FEM Software, is used for FEM analysis. Mode shapes and mode frequencies are obtained. Then results are compared with Euler-Bernoulli based on analytical calculation results which are done by MATLAB.



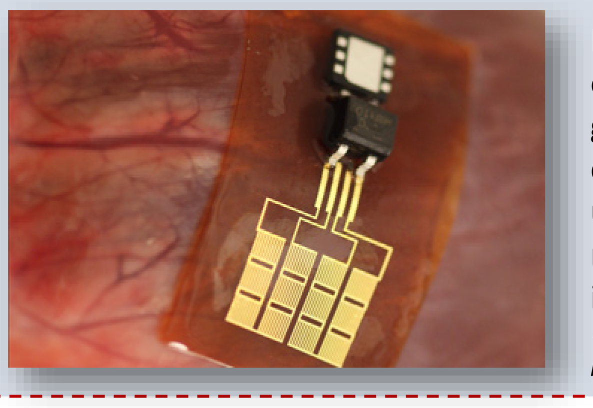
«Harvesting of vibration energy from mechanical operations or human motions is getting more popular due to its broad applications and power density (~ 200 μW/ cm3)»



SEM image of a microcantilever.



SEM image of MEMS piezoelectric vibration energy harvester.



Recently, Dagdeviren et al.* have announced new energy harvesting approach which aims to generating energy for pacemakers. They have developed a PZT cantilever based on device that uses the motion of heartbeats (as well as the motion of the lungs and diaphragm) to power an implanted pacemaker.

**Conformal piezoelectric energy harvesting and storage from motions of the heart, lung, and diaphragm." National Academy of Sciences (2014)*

Beam Theory and Mathematical Modelling

Euler-Bernoulli or thin beam theory is considered. General equation of motion or the beam structure is given below.

$$\frac{\partial^2 U(z,t)}{\partial t^2}\Gamma\rho + \frac{\partial^4 U(z,t)}{\partial z^4}EI = 0$$

Where $U(z,t)$is the displacement in the x-direction, ρ is the mass density, $\Gamma = wh$ is the cross-sectional area, E is Young's modulus and I is the geometric moment of inertia.

The solution $U(z,t) = U_n(z)\exp(-i2\pi f_n t)$, here f_n is the frequency of motion and where $n$to this differential equation is a harmonic that can be separated into a position dependent and a time-dependent term, n denotes the modal number.

$$EI\frac{\partial^4 U(z,t)}{\partial z^4} = k^4 U(z,t), \qquad k^4 = \frac{\omega^2 \Gamma\rho}{EI}$$

The solutions (eigenfunctions) to the simplified beam equation can be written in the form;

$$U_n(z) = A_n(\cos\kappa_n l - \cosh\kappa_n l) + B_n(\sin\kappa_n l - \sinh\kappa_n l)$$

The modal constants are determined from the boundary conditions. For a singly clamped beam (cantilever), the frequency equation becomes

$$1 + \cos(\kappa_n l)\cosh(\kappa_n l) = 0$$

Solutions for $\lambda_n = \kappa_n l$ are shown below:

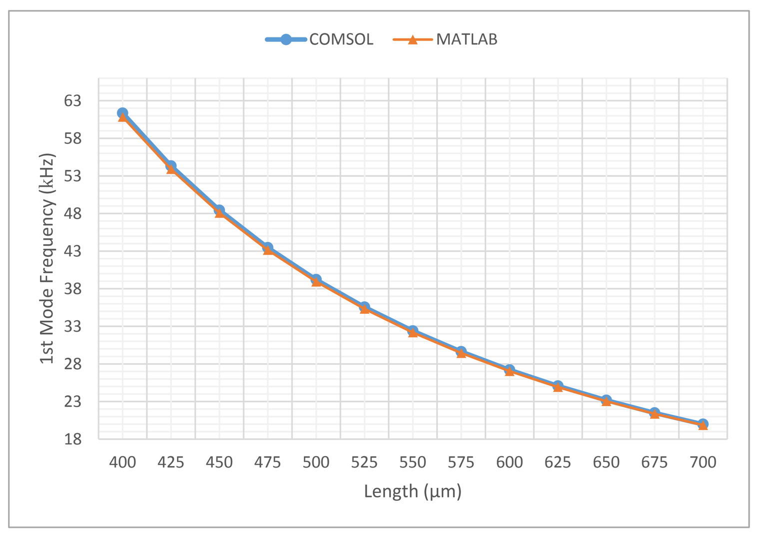
n	1	2	3	4	5	6
λ_n	1.875	4.694	7.855	10.996	14.137	17.279

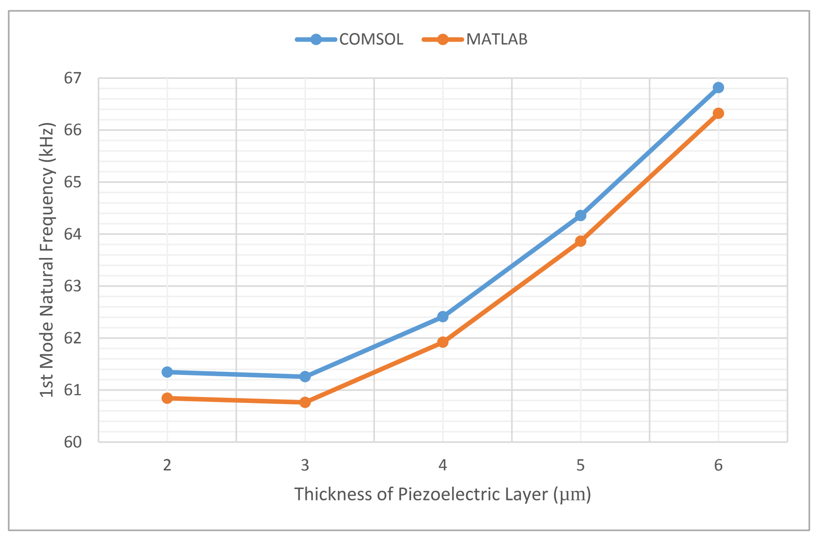
Results

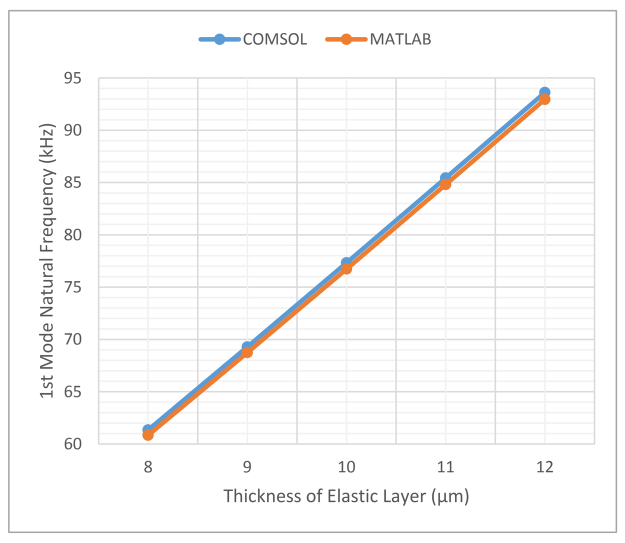
Cantilever properties are given below. For anlytical calculations, cantilever is broken 20 nodes. And then all global stiffness and mass matrices are solved using MATLAB reffering. Maximum displacement of cantilever is also calculated which is not presented here.

Layer / Material	Length (μm)	Width (μm)	Thickness (μm)	Elastic Modulus (GPa)	Density ($^{kg}/_{m^3}$)
Elastic / Silicon	400	64	8	170	2329
Piezoelectric / Lead Zirconate Titanate (PZT-5H)	400	64	2	63	7500

Relations between length,thickhness and mode frequencies are shown below.



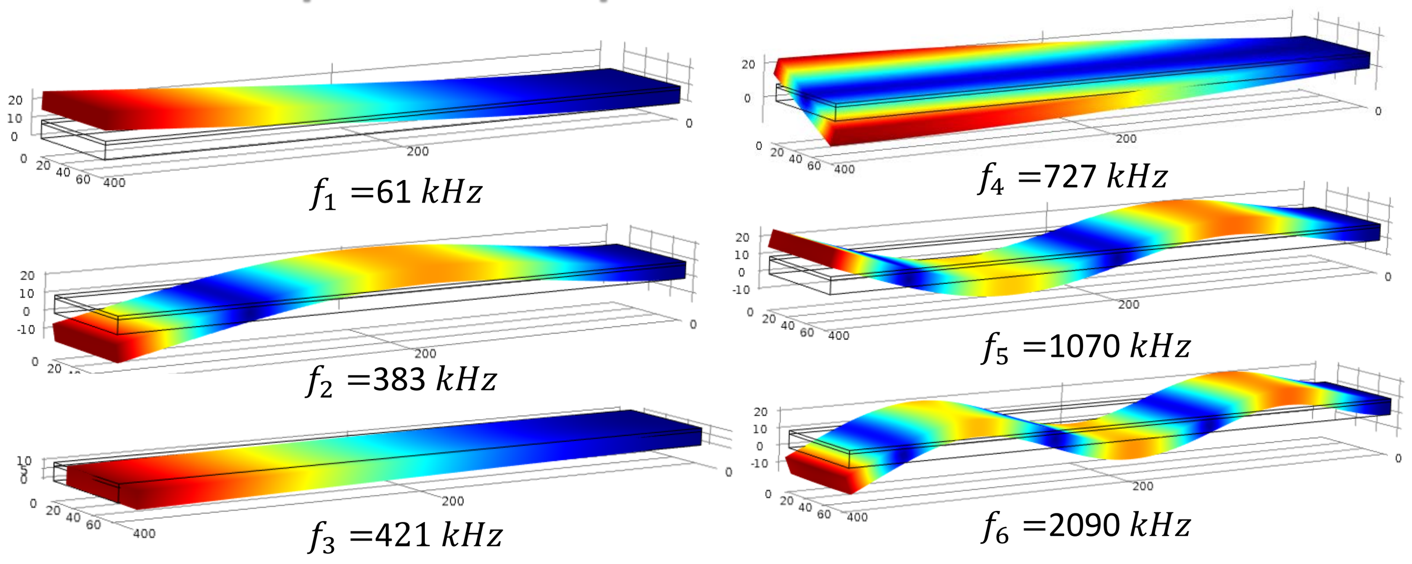


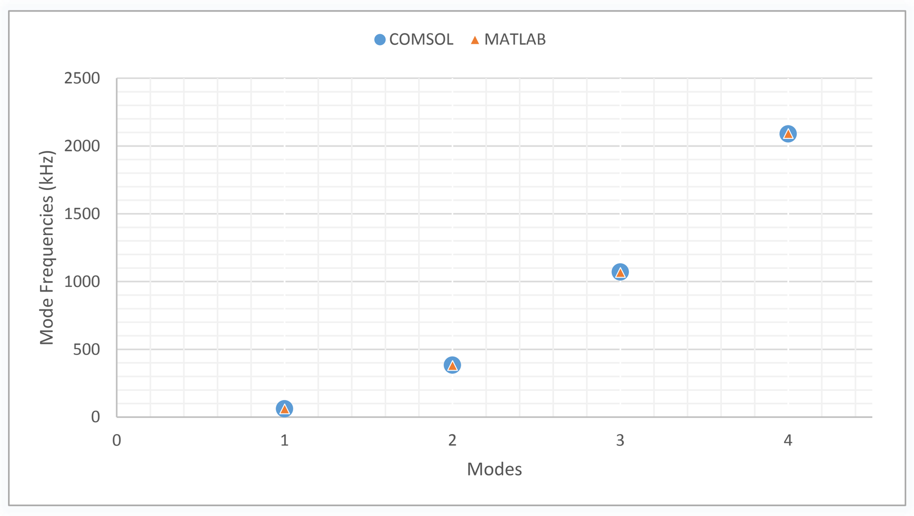


From left to right, the first graph shows Natural Frequency decreases while increasing length of cantilever. Those results are same as expected from natural frequency formula on the left column.

Second and third graphs show natural frequency increases while decreasing thichness of cantilever. Key point of between second and third graph is, piezoelectric layer is thinner and also softer than elastic layer. So thickness of the elastic layer effects the caltilever's frequency on a large scale.

Mod Shapes and Frequencies





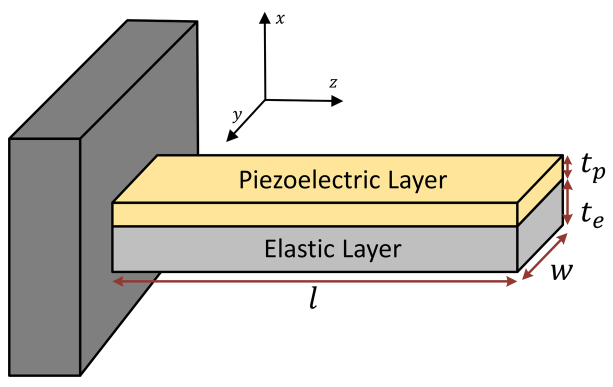
Second and upper mode frequencies decreases extremly comparing first mode because of the n mode number.

Physics of Microcantilever

Natural (resonance) frequency (f_n) and *spring constant* (k) of cantilever are expressed below.

$$f_n = \frac{\lambda_n^2}{2\pi}\sqrt{\frac{Eh^2}{12\rho l^4}}$$

Where E is Elastic Modulus, w is width and h is height of cantilever, t is *thickness* of the cantilever, l is *length* of the cantilever and ρ is *density* the cantilever.



Schematic of a simple cantilever geometry.

Natural frequency also can be rewritten in the following form:

$$f_n = \frac{\lambda_n^2}{2\pi}\sqrt{\frac{k}{3m}}$$

Where m is *mass* of the cantilever.

For a unimorph cantilever the spring mass is extracted according to *bending modulus* (D_p), *thickness* (t_p) and *Elastic Modulus* (E_p) of piezoelectric layer, *thickness* (t_e) and *Elastic Modulus* (E_e) of elastic layer:

$$D_p = \frac{E_p^2 t_p^4 + E_s^2 t_s^4 + 2E_p E_s t_p t_s(2t_p^2 + 2t_s^2 + 3t_p t_s)}{12(E_p t_p + E_s t_s)} \qquad k = \frac{3D_p w}{l^3}$$

Conclusions

This study presents simulation studies of unimorph piezoelectric beam structure for energy harvesting applications. The derivations of the mathematical equations are based on *Euler-Bernoulli* beam theory for analytical calculations. For unimorph structures natural frequency formula is written regarding to bending modulus and spring constant variables.

In this study we also have presented resonance frequencies and mode shapes for first 6 modes. As shown graphs before we discussed cantilever's dimentions and their effect on resonance frequency. Thickness of cantilever is directly proportional to natural frequency. However cantilever's length is inversly proportional to the natural frequency.

Natural frequency formula is only accurate for first mode frequency because of its linearity. Therefore FEM analysis are very important and necessary for accurate estimation.

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Acknowledgments

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